Range Analysis of Microcontroller Code Using Bit-Level Congruences

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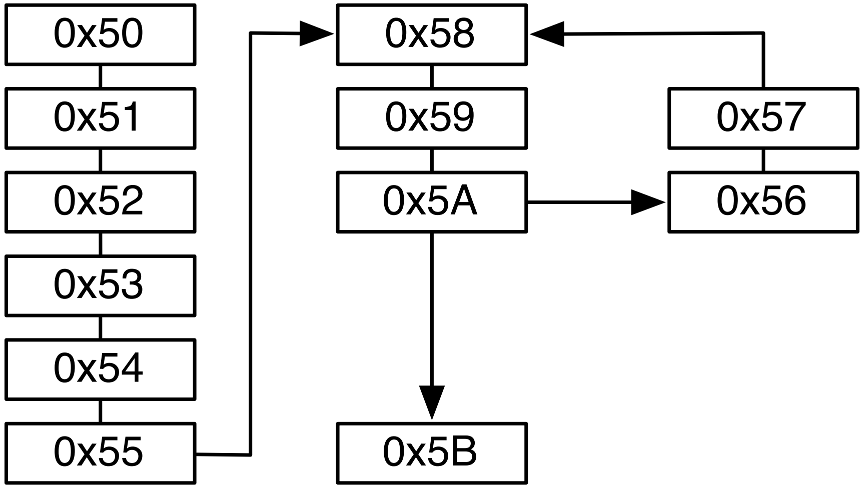
Motivating Example (1/2)

```
0x50: LDI r17 0
0x51: LDI r26 0
0x52: LDI r27 0
0x53: LDI r30 66
0x54: LDI r31 0
0x55: RJUMP 2
0x56: LPMPI r0 Z
0x57: STPI X r0
0x58: CPI r26 99
0x59: CPC r27 r17
0x5A: BRNE -5
0x5B: RET
```



- Loop copies three bytes from program memory into SRAM

Motivating Example (2/2)

```
0x50: LDI r17 0
0x51: LDI r26 0
0x52: LDI r27 0
0x53: LDI r30 66
0x54: LDI r31 0
0x55: RJUMP 2
0x56: LPMPI r0 Z
0x57: STPI X r0
0x58: CPI r26 99
0x59: CPC r27 r17
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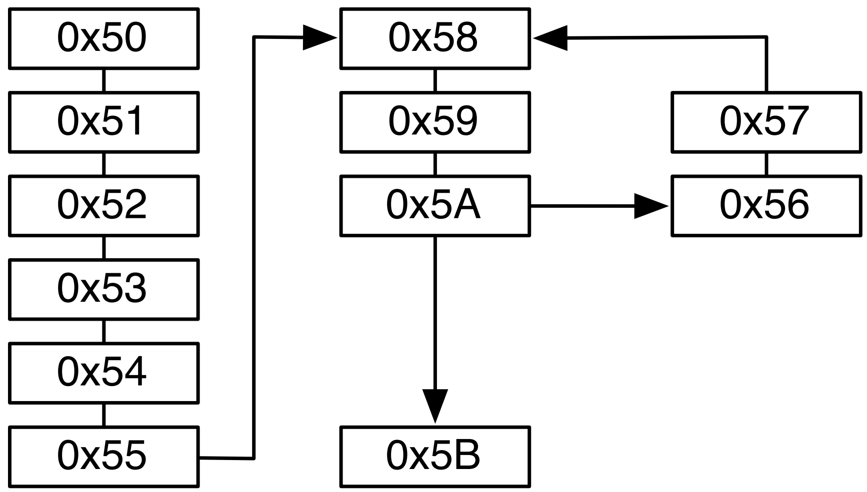

- Interval analysis derives $\texttt{X} \in [96, 98] \wedge \texttt{Z} \in \top$
- Insufficient for proving correctness
- Key idea: Combine with relational invariants (bit-level congruences) to prove $\texttt{X} \in [96, 98] \wedge \texttt{Z} \in [66, 68]$

Bit-Level Congruences

- Linear equations of the form $\sum_{i=0}^{n-1} \lambda_i \cdot v_i \equiv_m d$
  - v_i are bits
  - $\lambda_i \in \mathbb{Z}$ are coefficients
  - $m \in \mathbb{N}$ is a modulus
  - $d \in \mathbb{Z}$ is a constant
- Good to choose $m = 2^8$ for 8-bit microcontroller
- Can represent overflows that occur frequently

Example: Exclusive-Or

- Consider instruction `EOR r0 r1`
- Represented in Boolean logic:

$$\varphi \quad = \quad \wedge_{i=0}^{7} \mathbf{r0}'[i] \leftrightarrow \mathbf{r0}[i] \oplus \mathbf{r1}[i]$$

- Congruent abstraction of φ gives equations

$$\alpha_{\mathsf{cong}}(\varphi) \quad = \quad \wedge_{i=0}^{7} \left(128 \cdot \mathbf{r0}'[i] \equiv_{256} 128 \cdot \mathbf{r0}[i] + 128 \cdot \mathbf{r1}[i]\right)$$

- Computed using algorithm of [KS10]

Computing Congruent Abstractions of Entire Instruction Set

- Bit-blast concrete semantics of each instruction as defined in the specification
- Compute congruent abstraction using SAT solving
- Gives a set of transfer functions for each instruction in a program
- Need to be computed once, and can be reused afterwards

Deriving Congruent Invariants (1/3)

- Consider `EOR r0 r1; EOR r1 r0; EOR r0 r1;`
- Well-known idiom for register-swapping
- Goal: Relate inputs $\mathbf{r0}, \mathbf{r1}$ to outputs $\mathbf{r0}', \mathbf{r1}'$ to derive a suitable program invariant

Deriving Congruent Invariants (2/3)

- For the first two instructions, we have:

$$\bigwedge_{i=0}^{7} (128 \cdot \mathbf{r0}'[i] \equiv_{256} 128 \cdot \mathbf{r0}[i] + 128 \cdot \mathbf{r1}[i]) \wedge \bigwedge_{i=0}^{7} (\mathbf{r1}'[i] \equiv_{256} \mathbf{r1}[i]) \qquad (1)$$

$$\bigwedge_{i=0}^{7} (128 \cdot \mathbf{r1}'[i] \equiv_{256} 128 \cdot \mathbf{r0}[i] + 128 \cdot \mathbf{r1}[i]) \wedge \bigwedge_{i=0}^{7} (\mathbf{r0}'[i] \equiv_{256} \mathbf{r0}[i]) \qquad (2)$$

- Connect outputs of (1) to inputs of (2) and eliminate intermediate variables
- Elimination amounts to triangularization

Deriving Congruent Invariants (3/3)

- Invariant after second instruction:

$$\bigwedge_{i=0}^{7} (\mathbf{r1}'[i] \equiv_{256} \mathbf{r0}[i]) \wedge \bigwedge_{i=0}^{7} (128 \cdot \mathbf{r0}'[i] \equiv_{256} 128 \cdot \mathbf{r0}[i] + 128 \cdot \mathbf{r1}[i])$$

- Invariant after third instruction:

$$\bigwedge_{i=0}^{7} (\mathbf{r1}'[i] \equiv_{256} \mathbf{r0}[i]) \wedge \bigwedge_{i=0}^{7} (\mathbf{r0}'[i] \equiv_{256} \mathbf{r1}[i])$$

- That is, the implementation does what it is supposed to do

Initial Loop Revisited

- For the initial loop, this gives the invariant:

$$\mathbf{r26}' - \mathbf{r30}' \equiv_{256} 30 \wedge$$
$$\wedge_{i=0}^{7} (\mathbf{r17}'[i] \equiv_{256} 0 \wedge \mathbf{r27}'[i] \equiv_{256} 0 \wedge \mathbf{r31}'[i] \equiv_{256} 0)$$

- Difference between $\mathbf{r26}'$ and $\mathbf{r30}'$ as expected
- But no explicit information about range of $\mathbf{r30}'$
- Combine with intervals $\mathtt{X} \in [96, 98] \wedge \mathtt{Z} \in \top$ to prove that $\mathtt{X} \in [96, 98] \wedge \mathtt{Z} \in [66, 68]$

Reduction

- To do so, construct a map

$$\mathsf{reduce} : \mathsf{Int} \times \mathsf{Cong} \to \mathsf{Int} \times \mathsf{Cong}$$

with $\mathsf{reduce}(i, c) = (i', c')$ and

$$i' \sqsubseteq i \qquad c' \sqsubseteq c$$

- Intuitively, let information from one domain flow into the other computer narrower over-approximation

Stronger Intervals

- Convert constraint from intervals into logic:
$i = 96 \leq \mathbf{r26}' \leq 98 \wedge 0 \leq \mathbf{30}' \leq 255$

- Convert congruent invariant into logic, say, ψ
- Put $i \wedge \psi \wedge \mathbf{r30}'[7]$ and test satisfiability
- Unsatisfiable, hence $\mathbf{r30}' \leq 127$
- Put $i \wedge \psi \wedge \neg\mathbf{r30}'[7] \wedge \mathbf{r30}'[6]$ and test satisfiability
- Satisfiable, hence $\mathbf{r30}' \geq 64$
- Proceed with all bits to get $66 \leq \mathbf{r30}' \leq 68$

Stronger Congruences

- Apply similar idea to congruence equations
- Encode additional constraints from intervals into equation system
- And then project these additional constraints
- Gives stronger congruence equations, e.g.

$$\mathbf{r26'}[7] \equiv_{256} 0 \wedge \mathbf{r30'}[7] \equiv_{256} 0$$

- Technical details in the paper

Experiments: Transfer Function Synthesis

- Derived congruent abstractions for entire instruction set of ATMEL ATmega16 microcontroller
- Less than 1s for each instruction
- Some are exact (`EOR`, `INC`, `ADD`, etc.) others are not (`AND`, `OR`)
- By combining intervals and congruences, this loss of precision can be eased

Experiments: Computing Loop Invariants

- Invariant stabilized after 2 iterations
- Requiring 0.3s
- Computing join and eliminating variables is cubic in the number of bits
- Thus, it is a good idea to „slice“ variables not affected
- And use congruences where the interval analyzer loses precision

Experiments: Reducing Abstract Descriptions

- Reducing the intervals using SAT solving required 16 SAT instances
- Overall runtime using SAT4J amounts to 0.25s
- That is, two instances for each bit
- Reducing congruences requires computing upper-triangular form, approx. 0.1s

Related Work

- J. Brauer, T. Noll and B. Schlich: Interval Analysis of Microcontroller Code Using Abstract Interpretation of Hardware and Software (SCOPES 2010)
- A. King and H. Sondergaard: Automatic Abstraction for Congruences (VMCAI 2010)
- T. W. Reps, M. Sagiv and G. Yorsh: Symbolic Implementation of the Best Transformer (VMCAI 2004)
- P. Cousot and R. Cousot: Systematic Design of Program Analysis Frameworks (POPL 1979)
- M. Codish, V. Lagoon and P. J. Stuckey: Logic Programming with Satisfiability (TPLP 2008)

Discussion

- Deriving congruent invariants for binary/assembly code
- Combination with interval analysis
- A novel reduction operator that combines congruences and intervals
- Approach relies heavily on SAT-solving, which appears natural when reasoning about bits
- Allows to prove memory safety for many examples of binary code
- Integrated into [mc]square program verification tool

Thank you very much!